Neutrosophic Hypergraphs

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Abstract

In this paper, we introduce certain concepts, including neutrosophic hypergraph, line graph of neutrosophic hypergraph, dual neutrosophic hypergraph, tempered neutrosophic hypergraph and transversal neutrosophic hypergraph. We illustrate these concepts by several examples and investigate some of interesting properties.

Key-words: Neutrosophic hypergraph, Line graph of neutrosophic hypergraph, Dual neutrosophic hypergraph.
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1 Introduction

Fuzzy set was introduced by Zadeh [24] to solve difficulties in dealing with uncertainties. Since then the theory of fuzzy sets and fuzzy logic have been examined by many researchers to solve many real life problems, involving ambiguous and uncertain environment. Atanassov [4] introduced the concept of intuitionistic fuzzy sets as an extension of Zadeh's fuzzy set [24]. An intuitionistic fuzzy set can be viewed as an alternative approach when available information is not sufficient to define the impreciseness by the conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but intuitionistic fuzzy set is characterized by a membership(truth-membership) function and a non-membership(falsity-membership) function. Intuitionistic fuzzy set can deal only with incomplete information but not the indeterminate information and inconsistent information which commonly exist in certainty system. In intuitionistic fuzzy sets, indeterminacy is its hesitation part by default. Smarandache [19] initiated the concept of neutrosophic set in 1998. " It is the branch of philosophy which studied the origin, nature and scope of neutralities, as well as their interaction with different ideational spectra"[19]. A neutrosophic set is characterized by three components: truth-membership, indeterminacy-membership, and falsity-membership which are represented independently for dealing problems involving imprecise, indeterminacy and inconsistent data. In neutrosophic set, truth-membership, falsity-membership are independent, indeterminacy membership quantified explicitly, this assumption helps in a lot of situations such as information fusion when try to combine the data from different sensors. A Neutrosophic set is a general framework which generalizes the concept of fuzzy set, interval valued fuzzy set, and intuitionistic fuzzy set. Wang [22] presented the notion of single valued neutrosophic set(SVNS) and some set theoretic operators of neutrosophic set which is known as single valued neutrosophic set.

The hypergraph was introduced by Berge [9] and considered as a useful tool to analyze the structure of a system and to represent a partition, clustering and covering [6, 14, 18]. Fuzzy hypergraph was introduced by the Kaufmann [15]. Lee-kwang generalized the notion of fuzzy hypergraph and redefined it to be useful for fuzzy partition of a system. Chen [11] introduced interval-valued fuzzy hypergraphs. Akram and Dudek [1] investigated some properties of intuitionistic fuzzy hypergraph and gave applications of intuitionistic fuzzy hypergraph. The concepts of bipolar neutrosophic graphs and neutrosophic soft graphs are discussed in [3, 21]. In this paper, we introduce certain concepts, including neutrosophic hypergraph, line graph of neutrosophic hypergraph, dual neutrosophic hypergraph, tempered neutrosophic hypergraph and transversal neutrosophic hypergraph. We illustrate these concepts by several examples and investigate some of interesting properties.

A *single valued neutrosophic set* (SVNS) N in X is described by truth-membership function $T_N(x)$, indeterminacy-membership function $I_N(x)$ and falsity-membership function $F_N(x)$. For every element x in X, $T_N(x), I_N(x), F_N(x) \in [0,1]$, i.e., $N = \{\langle x, T_N, I_N(x), F_N(x)\rangle : x \in X\}$ and $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$. The hypergraph $H = (V, E^*)$ was defined by Berge [9] and is defined as a pair (V, E^*), where V is a finite set of nodes and E^* a finite family of subsets of V such that $V = \bigcup\limits_i E_i^*$. The number of vertices in an hyperedge E_i is called its cardinality; if $|E_i| = 1$ be a cycle on the element. If $|E_i| = 2$, for all i, the hypergraph becomes an ordinary graph. The rank of a hypergraph H is the maximium cardinality of any edge in hypergraph. If cardinality of all edges is same say k, then H is k-uniform hypergraph. The degree of a vertex v is the number of edges which contain the vertex v. A transversal T of hypergraph $H = (V, E^*)$ is a subset of V such that $T \cap E_i^* = \emptyset$ for all $E_i^* \in E^*$. The line graph (intersection graph) of simple hypergraph $H^* = (V, E^*)$ is the graph $L(H^*) = (V', E^{*\prime})$ such that $V' = E^*$ and $e_ie_j \in E^{*\prime}$ if and only if $E_i \cap E_j \neq \emptyset, i \neq j$. A transversal T is a minimal transversal of H if no proper subset of T is a transeversal of H. A hypergraph $H = (V; E_1^*, E_2^*, \cdots, E_m^*)$; $V = \{v_1, v_2, \cdots, v_n\}$ can be mapped to a hypergraph $H^* = (E^*; V_1, V_2, \cdots, V_n)$ whose vertices are $e_1, e_2, \cdots, e_m$ corresponding to $E_1, E_2, \cdots, E_m$, respectively. The hypergaph H^* is called the dual hypergraph of H. The incidence matrix of dual hypergraph is the transpose of hypergraph H, thus $(H^*)^* = H$. A pair $\mathbf{H} = (V, \mathbf{E})$ is a fuzzy hypergraph such that V is a finite set of vertices and $\mathbf{E}$ is a finite family of fuzzy sets on V, μ_i defined on $E_i \in \mathbf{E}$, $V = \bigcup\limits_i supp(\mu_i)$. A fuzzy hypergraph is μ tempered fuzzy hypergraph of $H = (V, E^*)$, if a fuzzy set $\mu : V \to (0, 1]$ exist and $\mathbf{E} = \{\lambda_{E_i} \mid e_i \in E^*\}$, where $\lambda_{E_i}(x) = \begin{cases} \min \mu(e) \mid e \in E_i, & \text{if } x \in E_i; \\ 0, & \text{otherwise.} \end{cases}$

2 Neutrosphic Hypergraphs

First we define here some fundamental notions.

Definition 2.1. The *support* set of a neutrosophic set $N = \{(x, T_N(x), I_N(x), F_N(x)) : x \in X\}$ is denoted by supp(N), defined by supp$(N) = \{x| \ T_N(x) \neq 0, I_N(x) \neq 0, F_N(x) \neq 0\}$. The support set of a neutrosophic set is a crisp set.

Definition 2.2. The *height* of a neutrosophic set $N = \{(x, T_N(x), I_N(x), F_N(x)) : x \in X\}$ is defined as $h(N) = (\sup_{x\in X} T_N(x), \sup_{x\in X} I_N(x), \inf_{x\in X} F_N(x))$. We call neutrosophic set N is *normal* then there exist at least one element x of X such that $T_N(x) = 1, I_N(x) = 1, F_N(x) = 0$.

Definition 2.3. Let $N = \{(x, T_N(x), I_N(x), F_N(x)) : x \in X\}$ be a neutrosophic set on X and let $\alpha, \beta, \gamma \in [0, 1]$ such that $\alpha + \beta + \gamma \leq 3$. Then the set $N_{(\alpha,\beta,\gamma)} = \{x \mid T_N(x) \geq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma\}$ is called $(\alpha, \beta, \gamma)-$level subset of N. $(\alpha, \beta, \gamma)-$level set is a crisp set.

Example 2.4. Let $X = \{a_1, a_2, a_3, a_4, a_5\}$. Then $N = \{(a_1, 0.5, 0.6, 0.3), (a_2, 0.3, 0.3, 0.5), (a_3, 0.2, 0.5, 0.7), (a_4, 0.5, 0.6, 0.2), (a_5, 0.4, 0.2, 0.6)\}$ is a neutrosophic subset of X. Clearly, $supp(N) = \{a_1, a_2, a_3, a_4, a_5\}$, $h(N) = (0.5, 0.6, 0.2)$. Let $(0.5, 0.3, 0.6) \in [0, 1]$, (0.5, 0.3, 0.6)-level set is $N_{(0.5,0.3,0.6)} = \{a_1, a_4\}$.

Definition 2.5. [5] A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $\mathbb{G} = (\mathbf{A}, \mathbf{B})$, where

(i) the functions $T_{\mathbf{A}} : V \to [0, 1]$, $I_{\mathbf{A}} : V \to [0, 1]$, and $F_{\mathbf{A}} : V \to [0, 1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $x_i \in V$, respectively, and

$$0 \leq T_{\mathbf{A}}(x_i) + I_{\mathbf{A}}(x_i) + F_{\mathbf{A}}(x_i) \leq 3 \text{ for all } x_i \in V, i = 1, 2, 3, \cdots, n.$$

(ii) the functions $T_{\mathbf{B}} : E \subseteq V \times V \to [0, 1]$, $I_{\mathbf{B}} : E \subseteq V \times V \to [0, 1]$, and $F_{\mathbf{B}} : E \subseteq V \times V \to [0, 1]$ are defined by

$$\begin{aligned} T_{\mathbf{B}}(x_i x_j) &\leq \min(T_{\mathbf{A}}(x_i), T_{\mathbf{A}}(x_j)) \\ T_{\mathbf{B}}(x_i x_j) &\leq \min(T_{\mathbf{A}}(x_i), T_{\mathbf{A}}(x_j)) \\ T_{\mathbf{B}}(x_i x_j) &\geq \max(T_{\mathbf{A}}(x_i), T_{\mathbf{A}}(x_j)) \end{aligned}$$

denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $x_i x_j \in E$, respectively, where

$$0 \leq T_{\mathbf{A}}(x_i x_j) + I_{\mathbf{A}}(x_i x_j) + F_{\mathbf{A}}(x_i x_j) \leq 3 \text{ for all } x_i x_j \in E, i = 1, 2, 3, \cdots, n.$$

We call $\mathbf{A}$ the single valued neutrosophic vertex set of V, $\mathbf{B}$ the single valued neutrosophic edge set of E, respectively. Note that the $\mathbf{B}$ is a symmetric single valued neutrosophic relation on $\mathbf{A}$.

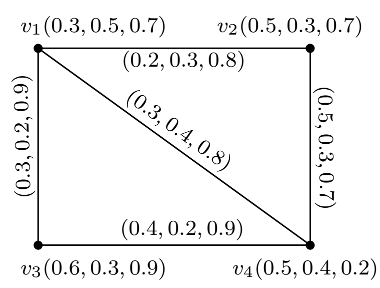

Figure 2.1: Single valued neutrosophic graph.

Definition 2.6. Let $\mathbb{G} = (A, B)$ be a SVN graph. An edge (x, y) of $\mathbb{G}$ is an *effective edge* if $T_B(x, y) = T_A(x) \wedge T_A(y)$, $I_B(x, y) = I_A(x) \wedge I_A(y)$ and $F_B(x, y) = F_A(x) \vee F_A(y)$.

Definition 2.7. Let $V = \{v_1, v_2, \cdots, v_n\}$ be a finite set of vertices and $\mathbb{E} = \{E_1, E_2, \cdots, E_m\}$ be a finite family of non-trivial neutrosophic subsets of the vertex V such that

$$V = \bigcup_i \mathrm{supp}(E_i), i = 1, 2, 3, \cdots, m,$$

where the edges E_i are neutrosophic subsets of V, $E_i = \{(v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}, E_i \neq \emptyset$, for $i = 1, 2, 3, \cdots, m$. $\mathbb{H} = (V, \mathbb{E})$ is an *neutrosophic hypergraph* on V, $\mathbb{E}$ is the family of neutrosophic hyperedges of $\mathbb{H}$ and V is the crisp vertex set of $\mathbb{H}$.

In neutrosophic hypergraphs two vertices v_1 and v_2 are adjacent if there exists an edge $E_i \in \mathbb{E}$ which have two vertices v_1 and v_2, i.e., $v_1, v_2 \in \mathrm{supp}(E_i)$. In neutrosophic hypergraphs $\mathbb{H}$, two vertices u and w are said to be connected if there exists a sequence $u = u_0, u_1, u_2, \cdots, u_n = v$ of vertices of H such that u_{i-1} is adjacent u_i for $i = 1, 2, \cdots, n$. When every pair of vertices in a neutrosophic hypergraph $\mathbb{H}$ are connected, $\mathbb{H}$ is connected. In a neutrosophic hypergraph two edges E_i and E_j are said to be adjacent if their intersection is non-empty, i.e., $supp(E_i) \cap supp(E_j) = \emptyset, i \neq j$. The order $|V|$ of a neutrosophic hypergraphs meant number of vertices and size $|\mathbb{E}|$ is number of edges of neutrosophic hypergraph. If $supp(E_i) = k$ for each $E_i \in \mathbb{E}$, then neutrosophic hypergraph $H = (V, \mathbb{E})$ is k-uniform neutrosophic hypergraph.

The element a_{ij} of the neutrosophic matrix represents the truth-membership (participation) degree, indeterminacy-membership degree and falsity-membership of v_i to E_j (that is $(T_{E_j}(v_i), I_{E_j}(v_i), F_{E_j}(v_i)))$. Since the diagram of neutrosophic hypergraph does not imply sufficiently the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of vertex to edges, we use incidence matrix $M_{\mathbb{H}}$ for the description of neutrosophic hyperedes.

Definition 2.8. The *height* of a neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, is denoted by $h(\mathbb{H})$, is defined by $h(\mathbb{H}) = \bigvee_i \{h(E_i)|\ E_i \in \mathbb{E}\}$.

Definition 2.9. Let $\mathbb{H} = (V, \mathbb{E})$ be a neutrosophic hypergraph, the *cardinality* of a neutrosophic hyperedge is the sum of truth-membership, indeterminacy-membership, and falsity-membership values of the vertices connects to an hyperedge, it is denoted by $|E_i|$. The *degree of a neutrosophic hyperedge*, $E_i \in \mathbb{E}$ is its cardinality, that is $d_{\mathbb{H}}(E_i) = |E_i|$. The *rank* of a neutrosophic hypergraph is the maximum cardinality of any neutrosophic hyperedge in $\mathbb{H}$, i.e., $\max\limits_{E_i \in \mathbb{E}} d_{\mathbb{H}}(E_i)$ and *anti rank* of a neutrosophic is the minimum cardinality of any neutrosophic hyperedge in $\mathbb{H}$, i.e., $\min\limits_{E_i \in \mathbb{E}} d_{\mathbb{H}}(E_i)$.

Definition 2.10. A *linear neutrosophic hypergraph* is a neutrosophic hypergraph in which every pair of distinct vertices of $\mathbb{H} = (V, \mathbb{E})$ is in at most one neutrosophic hyperedge of $\mathbb{H}$, i.e., $|supp(E_i) \cap supp(E_j)| \leq 1$ for all $E_i, E_j \in \mathbb{E}$. A $2-$uniform linear neutrosophic hypergraph is a neutrosophic graph.

Example 2.11. Condsider a neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $\mathbb{E} = \{E_1, E_2, E_3, E_4, E_5, E_6\}$, where $E_1 = \{(v_1, 0.3, 0.4, 0.6), (v_3, 0.7, 0.4, 0.4)\}$, $E_2 = \{(v_1, 0.3, 0.4, 0.6), (v_2, 0.5, 0.7, 0.6)\}$, $E_3 = \{(v_2, 0.5, 0.7, 0.6), (v_4, 0.6, 0.4, 0.8)\}$, $E_4 = \{(v_3, 0.7, 0.4, 0.4), (v_6, 0.4, 0.2, 0.7)\}$,

$E_5 = \{(v_3, 0.7, 0.4, 0.4), (v_5, 0.6, 0.7, 0.5)\}$, $E_6 = \{(v_5, 0.6, 0.7, 0.5), (v_6, 0.4, 0.2, 0.7)\}$, and $E_7 = \{(v_4, 0.6, 0.4, 0.8),$
$(v_6, 0.4, 0.2, 0.7)\}$.

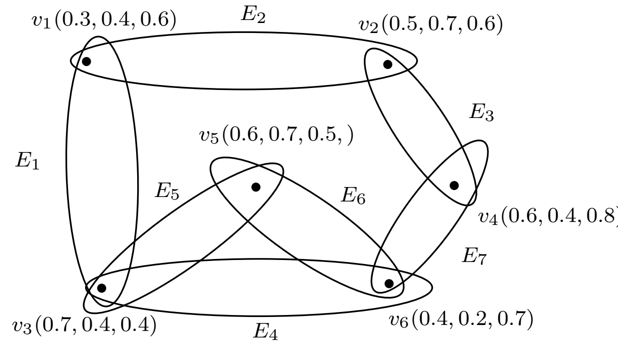

Figure 2.2: Neutrosophic hypergraph.

The neutrosophic hypergraph is shown in Figure. 2.2 and its incidence matrix $M_{\mathbb{H}}$ is given follows:

$M_{\mathbb{H}}$	E_1	E_2	E_3	E_4	E_5	E_6	E_7
v_1	$(0.3, 0.4, 0.6)$	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_2	$(0, 0, 0)$	$(0.5, 0.7, 0.6)$	$(0.5, 0.7, 0.6)$	$(0, 0., 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_3	$(0.7, 0.4, 0.4)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.7, 0.4, 0.4)$	$(0.7, 0.4, 0.4)$	$(0, 0, 0)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.4, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.4, 0.8)$
v_5	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.7, 0.5)$	$(0.6, 0.7, 0.5)$	$(0, 0, 0)$
v_6	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0.4, 0.2, 0.7)$

Definition 2.12. Let $\mathbb{H} = (V, \mathbb{E})$ be a neutrosophic hypergraph, the *degree* $d_{\mathbb{H}}(x)$ *of a vertex* x in $\mathbb{H}$ is $d_{\mathbb{H}}(v) = \sum\limits_{v \in E_i} (T_{E_i}(v), I_{E_i}(v), F_{E_i}(v))$, where E_i are the edges that contain the vertex v. The maximum degree of a neutrosophic hypergraph is $\triangle(\mathbb{H}) = \max\limits_{v \in V}(d_{\mathbb{H}}(v))$.

Definition 2.13. A neutrosophic hypergraph is said to be *regular neutrosophic hypergraph* in which all the vertices have same degree.

Proposition 2.14. *Let* $\mathbb{H} = (V, \mathbb{E})$ *be a neutrosophic hypergraph, then* $\sum\limits_{v \in V} d_{\mathbb{H}}(v) = \sum\limits_{E_i \in \mathbb{E}} d_{\mathbb{H}}(E_i)$.

Proof. Let $M_{\mathbb{H}}$ be the incidence matrix of neutrosophic hypergraph $\mathbb{H}$, then the sum of the degrees of each vertex $v_i \in V$ and the sum of degrees of each edge $E_i \in \mathbb{E}$ are equal. We obtain $\sum\limits_{v \in V} d_{\mathbb{H}}(v) = \sum\limits_{E_i \in \mathbb{E}} d_{\mathbb{H}}(E_i)$. $\square$

Definition 2.15. The *strength* η *of a neutrosophic hyperedge* E_i is the minimum of truth-membership and indeterminacy-membership values and maximum falsity-membership in the edge E_i, i.e.,

$$\eta(E_i) = \{\min_{v_j \in E_i}(T_{E_i}(v_j) \mid T_{E_i}(v_j) > 0), \min_{v_j \in E_i}(I_{E_i}(v_j) \mid I_{E_i}(v_j) > 0), \max_{v_j \in E_i}(F_{E_i}(v_j) \mid F_{E_i}(v_j) > 0)\}.$$

The strength of an edge in neutrosophic hypergraph interpreters that the edge E_i group elements having participation degree at least $\eta(E_i)$.

Example 2.16. Consider neutrosophic hypergraph as shown in Figure. 2.2, the height of H is $h(H) = (0.7, 0.7, 0.4)$, the strength of each edge is $\eta(E_1) = (0.3, 0.4, 0.6), \eta(E_2) = (0.3, 0.4, 0.6), \eta(E_3) = (0.5, 0.4, 0.8)$,
$\eta(E_4) = (0.4, 0.2, 0.7), \eta(E_5) = (0.6, 0.4, 0.5), \eta(E_6) = (0.4, 0.2, 0.7)$ and $\eta(E_7) = (0.4, 0.2, 0.8)$, respectively. The edges with high strength are called the strong edges because the interrelation (cohesion) in them is strong. Therefore, E_5 is stronger than each E_i, for $i = 1, 2, 3, 4, 6, 7$.
If we assign $\eta(E_i) = (T_{\eta(E_i)}, I_{\eta(E_i)}, F_{\eta(E_i)})$ to each clique in neutrosophic graph mapped to an edge E_i in neutrosophic hypergraph, we obtain a neutrosophic graph which represents subset with grouping strength(interrelationship).

$M_{\mathbb{H}}$	E_1	E_2	E_3	E_4	E_5	E_6	E_7
v_1	$(0.3, 0.4, 0.6)$	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_2	$(0, 0, 0)$	$(0.3, 0.4, 0.6)$	$(0.5, 0.4, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_3	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0.6, 0.4, 0.5)$	$(0, 0, 0)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0, 0, 0)$	$(0.5, 0.4, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.8)$
v_5	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.4, 0.5)$	$(0.4, 0.2, 0.7)$	$(0, 0, 0)$
v_6	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0.4, 0.2, 0.8)$

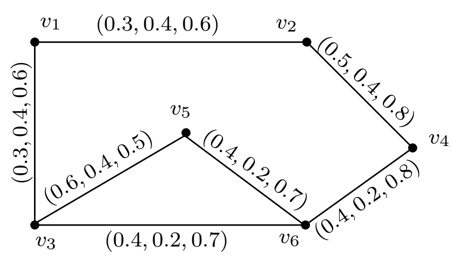

Figure 2.3: Corresponding neutrosophic graph.

We see that a neutrosophic graph can be associated with a neutrosophic hypergraph, a hyperedge with its strength η in the neutrosophic hypergraph is mapped to a clique in the neutrosophic graph, all edges in the clique have the same strength. Figure. 2.3 shows that corresponding neutrosophic graph to the neutrosophic hypergraph $\mathbb{H}$ in Fig. 2.2. In corresponding neutrosophic graph, the numbers attached to the edges represents the truth-membership, indeterminacy-membership and falsity membership of the edges.

Proposition 2.17. *Neutrosophic graphs and neutrosophic digraphs are special cases of the neutrosophic hypergraphs.*

Definition 2.18. A neutrosophic set $N = \{(x, T_N(x)), I_N(x), F_N(x) \mid x \in X\}$ is an elementary neutrosophic set if N is single valued on *supp*(N). An elementary neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ is a neutrosophic hypergraph in which each element of $\mathbb{E}$ is elementary.

Definition 2.19. A neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ is called *simple neutrosophic hypergraph* if $\mathbb{E}$ has no repeated neutrosophic hyperedges and whenever $E_i, E_j \in \mathbb{E}$ and $T_{E_i} \leq$

$T_{E_j}, I_{E_i} \leq I_{E_j}, F_{E_i} \geq F_{E_j}$, then $T_{E_i} = T_{E_j}, I_{E_i} = I_{E_j}, F_{E_i} = F_{E_j}$. A neutrosophic hypergraph is called *support simple*, if whenever $E_i, E_j \in \mathbb{E}, E_i \subset E_j$ and supp(E_i) =suppE_j, then $E_i = E_j$. A neutrosophic hypergraph is called *strongly support simple* if whenever $E_i, E_j \in \mathbb{E}$, and supp(E_i) =suppE_j, then $E_i = Ej$.

Definition 2.20. Let $\mathbb{H} = (V, \mathbb{E})$ be a neutrosophic hypergraph. Let $\alpha, \beta, \gamma \in [0, 1]$ and $\mathbb{E}^{(\alpha,\beta,\gamma)} = \{E_i^{(\alpha,\beta,\gamma)} \mid E_i \in \mathbb{E}\}$ and $V^{(\alpha,\beta,\gamma)} = \bigcup\limits_{E_i \in \mathbb{E}} E_i^{(\alpha,\beta,\gamma)}$. $\mathbb{H}^{(\alpha,\beta,\gamma)} = (V^{(\alpha,\beta,\gamma)}, \mathbb{E}^{(\alpha,\beta,\gamma)})$ is the (α, β, γ)-level hypergraph of $\mathbb{H} = (V, \mathbb{E})$, where $\mathbb{E}^{(\alpha,\beta,\gamma)} \neq \emptyset$. $\mathbb{H}^{(\alpha,\beta,\gamma)}$ is a crisp hypergraph.

Remark 2.21.

1. A neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ is a neutrosophic graph (with loops) if and only if $\mathbb{H}$ is elementary, each edge has two (or one) element support and support simple.
2. For a simple neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, (α, β, γ)-level hypergraph $\mathbb{H}^{(\alpha,\beta,\gamma)}$ may or may not be simple neutrosophic hypergraph. For any two neutrosophic hyperedges $E_i, E_j \in \mathbb{E}$, it is possible $E_i^{(\alpha,\beta,\gamma)} = E_j^{(\alpha,\beta,\gamma)}$ for $E_i \neq E_j$.
3. $\mathcal{H}$ and $\mathcal{H}'$ are two families of simple hypergraphs(sets) formed by $(\alpha, \beta, \gamma)-$levels of neutrosophic hypergraphs. $\mathcal{H}$ and $\mathcal{H}'$ have an important relationship in common, for each set $H \in \mathcal{H}$ there exist a set $H' \in \mathcal{H}'$ which is superset of H. We say that $\mathcal{H}'$ absorbs $\mathcal{H}$, i.e., $\mathcal{H} \sqsubseteq \mathcal{H}'$. Since it is possible $\mathcal{H}'$ absorbs $\mathcal{H}$ while $\mathcal{H}' \cap \mathcal{H} = \emptyset$, then $\mathcal{H} \subseteq \mathcal{H}'$ implies $\mathcal{H} \sqsubseteq \mathcal{H}'$, but on the other hand it is usually false that is, if $\mathcal{H} \sqsubseteq \mathcal{H}'$ and $\mathcal{H} \neq \mathcal{H}'$, then $\mathcal{H} \sqsubset \mathcal{H}'$.

Definition 2.22. Let $\mathbb{H} = (V, \mathbb{E})$ be a neutrosophic hypergraph, and let $h(\mathbb{H}) = (r, s, t)$, $\mathbb{H}^{(r_i,s_i,t_i)} = (V^{(r_i,s_i,t_i)}, \mathbb{E}^{(r_i,s_i,t_i)})$ be the $(r_i, s_i, t_i)-$level hypergraphs of $\mathbb{H}$. The sequence of real numbers $\{(r_1, s_1, t_1),$
$(r_2, s_2, t_2), \cdots, (r_n, s_n, t_n)\}$, $0 < r_n < r_{n-1} < \cdots < r_1 = r$, $0 < s_n < s_{n-1} < \cdots < s_1 = s$, and $t_n > t_{n-1} > \cdots > t_1 = t > 0$, which satisfies the properties:

1. if $r_{i+1} < r' < r_i, s_{i+1} < r' < s_i, t_{i+1} > t' > t_i (t_i < t' < t_{i+1})$, then $\mathbb{E}^{(r',s',t')} = \mathbb{E}^{(r_i,s_i,t_i)}$,
2. $\mathbb{E}^{(r_i,s_i,t_i)} \sqsubset \mathbb{E}^{(r_i+1,s_i+1,t_i+1)}$,

is fundamental sequence of neutrosophic hypergraph $\mathbb{H}$, denoted by $\mathrm{F}(\mathbb{H})$ and the set of $(r_i, s_i, t_i)-$level hypergraphs $\{\mathbb{H}^{(r_1,s_1,t_1)}, \mathbb{H}^{(r_2,s_2,t_2)}, \cdots, \mathbb{H}^{(r_n,s_n,t_n)}\}$ is known as *core* hypergraphs of neutrosophic hypergraph $\mathbb{H}$, and is denoted by $\mathrm{C}(\mathbb{H})$.

If $r_1 < r \leq 1, s_1 < s \leq 1, 0 \geq t < t_1$, then $\mathbb{E}^{(r,s,t)} = \{\emptyset\}$ and $\mathbb{H}^{(r,s,t)}$ does not exist.

Definition 2.23. Suppose $\mathbb{H} = (V, \mathbb{E})$ is a neutrosophic hypergraph with $\mathrm{F}(\mathbb{H}) = \{(r_1, s_1, t_1), (r_2, s_2, t_2),$ $\cdots, (r_n, s_n, t_n)\}$ and $r_{n+1} = 0, s_{n+1} = 0, t_{n+1} = 0$. $\mathbb{H}$ is *sectionally elementary* if every element $E_i \in \mathbb{E}$ and each $(r_i, s_i, t_i) \in \mathrm{F}(\mathbb{H})$, $\mathbb{E}_i^{(r_i,s_i,t_i)} = \mathbb{E}_i^{(r,s,t)}$ for all $(r, s, t) \in \big((r_{i+1}, s_{i+1}, t_{i+1}), (r_i, s_i, t_i)\big]$.

Definition 2.24. Suppose that $\mathbb{H} = (V, \mathbb{E})$ and $H' = (V', \mathbf{E}')$ are neutrosophic hypergraphs. $\mathbb{H}$ is called a *partial neutrosophic hypergraph* of $\mathbb{H}'$ if $\mathbb{E} \subseteq \mathbb{E}'$. If $\mathbb{H}$ is partial neutrosophic hypergraphs of $\mathbb{H}'$, we write $\mathbb{H} \subseteq \mathbb{H}'$. If $\mathbb{H}$ is partial neutrosophic hypergraph of $\mathbb{H}'$ and $\mathbb{E} \subset \mathbb{E}'$, then we denoted as $\mathbb{H} \subset \mathbb{H}'$.

Example 2.25. Consider the neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, where $V = \{v_1, v_2, v_3, v_4\}$ and $\mathbb{E} = \{E_1, E_2, E_3, E_4, E_5\}$, which is represented by the following incidence matrix:

$M_{\mathbb{H}}$	E_1	E_2	E_3	E_4	E_5
v_1	$(0.7, 0.6, 0.5)$	$(0.9, 0.8, 0.1)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.3, 0.3)$
v_2	$(0.7, 0.6, 0.5)$	$(0.9, 0.8, 0.1)$	$(0.9, 0.8, 0.1)$	$(0.7, 0.6, 0.5)$	$(0, 0, 0)$
v_3	$(0, 0, 0)$	$(0, 0, 0)$	$(0.9, 0.8, 0.1)$	$(0.7, 0.6, 0.5)$	$(0.4, 0.3, 0.3)$
v_4	$(0, 0, 0)$	$(0.4, 0.3, 0.3)$	$(0, 0, 0)$	$(0.4, 0.3, 0.3)$	$(0.4, 0.3, 0.3)$

Clearly, $h(\mathbb{H}) = (0.9, 0.8, 0.1)$, $E_1^* = \mathbb{E}^{(0.9,0.8,0.1)} = \{\{v_1, v_2\}, \{v_2, v_3\}\} = \mathbb{E}^{(0.7,0.6,0.5)}$ and $E_2^* = \mathbb{E}^{(0.4,0.3,0.3)} = \{\{v_1, v_2\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_1, v_3, v_4\}\}$. Therefore, fundamental sequence is $\mathrm{F}(\mathbb{H}) = \{(r_1, s_1, t_1) = (0.9, 0.8, 0.1), (r_2, s_2, t_2) = (0.4, 0.3, 0.3)\}$ and the set of core hypergraph is $\mathrm{C}(\mathbb{H}) = \{\mathbb{H}^{(0.9,0.8,0.1)} = (V_1, E_1^*), \mathbb{H}^{(0.4,0.3,0.3)} = (V_2, E_2^*)\}$. Note that, $\mathbb{E}^{(0.9,0.8,0.1)} \sqsubseteq \mathbb{E}^{(0.4,0.3,0.3)}$ and $\mathbb{E}^{(0.9,0.8,0.1)} \neq \mathbb{E}^{(0.4,0.3,0.3)}$. As $E_5 \subseteq E_2$, $\mathbb{H}$ is not simple neutrosophic hypergraph but $\mathbb{H}$ is support simple. In neutrosophic graph $\mathbb{H} = (V, \mathbb{E})$, $\mathbb{E}_1^{(r,s,t)} \neq \mathbb{E}_1^{(0.9,0.8,0.1)}$ for $(r, s, t) = (0.7, 0.6, 0.5)$, $\mathbb{H}$ is not sectionally elementary.

The partial neutrosophic hypergraphs, $\mathbb{H}' = (V', \mathbb{E}')$, where $\mathbb{E}' = \{E_2, E_3, E_4, E_1\}$ is simple, $\mathbb{H}'' = (V'', \mathbb{E}'')$, where $\mathbb{E}'' = \{E_2, E_3, E_5\}$ is sectionally elementary, and $\mathbb{H}''' = (V''', \mathbb{E}''')$, where $\mathbb{E}''' = \{E_1, E_3, E_5\}$ is elementary.

Definition 2.26. A *ordered* neutrosophic hypergraphs is a neutrosophic hypergraph said in which $\mathrm{C}(\mathbb{H})$ is ordered, i.e., if $\mathrm{C}(\mathbb{H}) = \{\mathbb{H}^{(r_1,s_1,t_1)}, \mathbb{H}^{(r_2,s_2,t_2)}, \cdots, \mathbb{H}^{(r_n,s_n,t_n)}\}$, then $\mathbb{H}^{(r_1,s_1,t_1)} \subseteq \mathbb{H}^{(r_2,s_2,t_2)} \subseteq \cdots \subseteq \mathbb{H}^{(r_n,s_n,t_n)}$. If $\mathrm{C}(\mathbb{H})$ is ordered and if whenever $E^* \in E_{j+1}^* \setminus E_j^*$, then $E^* \nsubseteq V_j$ then neutrosophic hypergraph $\mathbb{H}$ is *simply ordered.*

Proposition 2.27. *If $\mathbb{H} = (V, \mathbb{E})$ is an elementary neutrosophic hypergraph, then $\mathbb{H}$ is ordered. Also, if $\mathbb{H} = (V, \mathbb{E})$ is an ordered neutrosophic hypergraph with $\mathrm{C}(\mathbb{H}) = \{\mathbb{H}^{(r_1,s_1,t_1)}, \mathbb{H}^{(r_2,s_2,t_2)}, \cdots, \mathbb{H}^{(r_n,s_n,t_n)}\}$ and if $\mathbb{H}^{(r_n,s_n,t_n)}$ is simple, then $\mathbb{H}$ is elementary.*

Definition 2.28. A neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ is called a E^t *tempered neutrosophic hypergraph* of $H^* = (V, E^*)$ if there is a crisp hypergraph $H^* = (V, E^*)$ and neutrosophic set E^t is defined on V, where $T_{E^t} : V \to (0, 1]$, $I_{E^t} : V \to (0, 1]$, and $F_{E^t} : V \to (0, 1]$ such that $\mathbb{E} = \{C_E \mid E \in E^*\}$, where

$$T_{C_E}(x) = \begin{cases} \wedge\{T_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{C_E}(x) = \begin{cases} \wedge\{I_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$$

$$F_{C_E}(x) = \begin{cases} \vee\{F_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$$

We let $E^t \otimes H^*$ denote the E^t tempered neutrosophic hypergraph of $H^* = (V, E^*)$ and neutrosophic set E^t.

Example 2.29. Consider the neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, where $V = \{v_1, v_2, v_3, v_4\}$ and $\mathbb{E} = \{E_1, E_2, E_3, E_4\}$, which is represented by the following incidence matrix:

$M_{\mathbb{H}}$	E_1	E_2	E_3	E_4
v_1	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0.1, 0.4, 0.5)$	$(0.3, 0.4, 0.5)$
v_2	$(0, 0, 0)$	$(0.1, 0.4, 0.3)$	$(0, 0, 0)$	$(0.3, 0.4, 0.5)$
v_3	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0.1, 0.4, 0.3)$	$(0.1, 0.4, 0.5)$	$(0, 0, 0)$

Define $E^t = \{(v_1, 0.3, 0.4, 0.5), (v_2, 0.6, 0.5, 0.2), (v_3, 0.5, 0.4, 0.6), (v_4, 0.1, 0.4, 0.3)\}$. Note that $T_{\{v_1,v_3\}}(v_1) = T_{E^T}(v_1) \wedge T_{E^T}(v_3) = 0.3, I_{\{v_1,v_3\}}(v_1) = I_{E^t}(v_1) \wedge I_{E^t}(v_3) = 0.4, F_{\{v_1,v_3\}}(v_1) = F_{E^T}(v_1) \vee F_{E^T}(v_3) = 0.6$, and $T_{\{v_1,v_3\}}(V_3) = T_{E^T}(v_3) \wedge T_{E^T}(v_1) = 0.3, I_{\{v_1,v_3\}}(v_3) = I_{E^t}(v_3) \wedge I_{E^t}(v_1) = 0.4, F_{\{v_1,v_3\}}(V_3) = F_{E^T}(v_3) \vee F_{E^T}(v_1) = 0.6$, then $C_{\{v_1,v_3\}} = E_1$. Also $C_{\{v_2,v_4\}} = E_2$, $C_{\{v_1,v_4\}} = E_3$, $C_{\{v_1,v_2\}} = E_4$. Thus $\mathbb{H}$ is E^t tempered.

Theorem 2.30. *A neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ is a E^t tempered neutrosophic hypergraph of H^* if and only if $\mathbb{H}$ is elementary, support simple and simple ordered.*

Proof. Suppose $\mathbb{H} = (V, \mathbb{E})$ is a E^t tempered neutrosophic hypergraph of H^*. Obviously, $\mathbb{H}$ is elementary and support simple. We have to prove that $\mathbb{H}$ is simply ordered. Let $\mathrm{C}(\mathbb{H}) = \{\mathbb{H}^{(r_1,s_1,t_1)} = (V_1, E_1^*), \mathbb{H}^{(r_2,s_2,t_2)} = (V_2, E_2^*), \cdots, \mathbb{H}^{(r_n,s_n,t_n)} = (V_n, E_n^*)\}$. Since $\mathbb{H}$ is elementary, it follows from Proposition. 3.27 $\mathbb{H}$ is ordered. Suppose there exist $E \in E_{i+1}^* \setminus E_i^*$ and $v \in E$ such that $T_E(v) = r_{i+1}, I_E(v) = s_{i+1}$, and $F_E(v) = t_{i+1}$. Since $T_E(v) = r_{i+1} < r_i, I_E(v) = s_{i+1} < s_i$, and $F_E(v) = t_{i+1} > t_i$, it follows that $v \notin V_i$ and $E \nsubseteq V_i$, hence $\mathbb{H}$ is simply ordered.

Conversely, suppose $\mathbb{H} = (V, \mathbb{E})$ is elementary, support simple and simply ordered. For $\mathrm{C}(\mathbb{H}) = \{\mathbb{H}^{(r_1,s_1,t_1)} = (V_1, E_1^*), \mathbb{H}^{(r_2,s_2,t_2)} = (V_2, E_2^*), \cdots, \mathbb{H}^{(r_n,s_n,t_n)} = (V_n, E_n^*)\}$, fundamental sequence is $\mathrm{F}(\mathbb{H}) = \{(r_1, s_1, t_1), (r_2, s_2, t_2), \cdots, (r_n, s_n, t_n)\}$ with $0 < r_n < r_{n-1} < \cdots < r_1$, $0 < s_n < s_{n-1} < \cdots < s_1$, and $0 < t_1 < t_2 < \cdots < t_n$. $\mathbb{H}^{(r_n,s_n,t_n)} = (V_n, E_n^*)$ and neutrosophic set E^t on V_n defined by $T_{E^t}(v) = \begin{cases} r_1, & \text{if } v \in V_1; \\ r_i, & \text{if } v \in V_i \setminus V_{i-1}, i = 2, 3, \cdots, n. \end{cases}$

$I_{E^t}(v) = \begin{cases} s_1, & \text{if } v \in V_1; \\ s_i, & \text{if } v \in V_i \setminus V_{i-1}, i = 2, 3, \cdots, n. \end{cases}$

$F_{E^t}(v) = \begin{cases} t_1, & \text{if } v \in V_1; \\ t_i, & \text{if } v \in V_i \setminus V_{i-1}, i = 2, 3, \cdots, n. \end{cases}$

We show that $\mathbb{E} = \{C_E \mid E \in E_n^*\}$,where

$T_{C_E}(x) = \begin{cases} \wedge\{T_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$

$I_{C_E}(x) = \begin{cases} \wedge\{I_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$

$F_{C_E}(x) = \begin{cases} \vee\{F_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$

Let $E \in E_n^*$. Since $\mathbb{H}$ is elementary and support simple there is a unique neutrosophic hyperedge E_j in $\mathbb{E}$ having support $E \in E_n^*$. We have to show that E^t tempered neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ determined by the crisp graph $H_n^* = (V_n, E_n^*)$, i.e., $C_{E \in E_n^*} = E_i, i = 1, 2, \cdots, m$.
As all neutrosophic hyperedges are elementary and $\mathbb{H}$ is support simple, then different edges have different supports, that is $h(E_j)$ is equal to some member (r_i, s_i, t_i) of $\mathrm{F}(\mathbb{H})$. Consequently, $E \subseteq V_i$ and if $i > 1$, then $E \in E_i^* \setminus E_{i-1}^*$, $T_E(v) \geq r_i, I_E(v) \geq s_i$, and $F_E(v) \leq t_i$ for some $v \in E$.

Since $E \subseteq V_i$,, we claims that $T_{E^t}(v) = r_i, I_{E^t}(v) = s_i, F_{E^t}(v) = t_i$ for some $v \in E$, if not then $T_{E^t}(v) \geq r_{i-1}, I_{E^t}(v) \geq s_{i-1}, F_{E^t}(v) \leq t_{i-1}$ for all $v \in E$ which implies $E \subseteq V_{i-1}$ and since $\mathbb{H}$ is simply ordered, $E \in E_i^* \setminus E_{i-1}^*$, then $E \nsubseteq V_{i-1}$, a contradiction. Thus $C_E = E_i, i = 1, 2, \cdots, m$, by the definition of C_E. □

Corollary 2.31. *Suppose* $\mathbb{H} = (V, \mathbb{E})$ *is a simply ordered neutrosophic hypergraph and* $\mathrm{F}(\mathbb{H}) = \{(r_1, s_1, t_1),$
$(r_2, s_2, t_2), \cdots, (r_n, s_n, t_n)\}$. *For a simple hypergraph* $\mathbb{H}^{(r_n, s_n, t_n)}$, *there is a partial neutrosophic hypergraph* $\mathbb{H}' = (V, \mathbb{E}')$ *of* $\mathbb{H} = (V, \mathbb{E})$ *such that following statements hold:*

1. $\mathbb{H}'$ *is a* E^t *tempered neutrosophic hypergraph of* $\mathbb{H}^{(r_n, s_n, t_n)}$.

2. $\mathrm{F}(\mathbb{H}') = \mathrm{F}(\mathbb{H})$ *and* $\mathrm{C}(\mathbb{H}') = \mathrm{C}(\mathbb{H})$.

Proof. Since $\mathbb{H}$ is simple ordered, then $\mathbb{H}$ is an elementary neutrosophic hypergraph. We obtain the partial neutrosophic hypergraph $\mathbb{H}' = (V, \mathbb{E}')$ of $\mathbb{H} = (V, \mathbb{E})$ by removing all edges from $\mathbb{E}$ that are properly contained in another edge of $\mathbb{H}$, where $\mathbb{E}' = \{E_i \in \mathbb{E} \mid \text{if } E_i \subseteq E_j \text{ and } E_j \in \mathbb{E}, \text{ then } E_i = E_j\}$.

Since $\mathbb{H}^{(r_n, s_n, t_n)}$ is simple hypergraph in which all edges are elementary, any edge in $\mathbb{H}$ subset of another edge then both edges have the same support. So $\mathrm{F}(\mathbb{H}') = \mathbb{H}$ and $\mathrm{C}(\mathbb{H}') = \mathrm{C}(\mathbb{H})$. By the definition of $\mathbb{E}'$, $\mathbb{H}'$ is elementary, support simple. Thus by the Theorem 3.30 $\mathbb{H}'$ is a E^t tempered neutrosophic hypergraph. □

We now define neutrosophic line graph and neutrosophic line graph of a neutrosophic hypergraph.

Definition 2.32. Let $L(G) = (C, D)$ be a line graph of crisp graph $G = (V, E^*)$, where $C = \{\{x\} \cup \{u_x, v_x\} \mid x \in E^*, u_x, v_x \in V, x = u_x v_x\}$ and $D = \{S_x S_y \mid S_x \cap S_y \neq \emptyset, x, y \in E^*, x \neq y\}$ and where $S_x = \{\{x\} \cup \{u_x, v_x\}\}, x \in E^*$. Let $\mathbb{G} = (A_1, B_1)$ be a neutrosophic graph with underlying set V. Let A_2 be the neutrosophic vertex set of C, B_2 be the neutrosophic edge set of D. The *neutrosophic line graph* of $\mathbb{G}$ is a neutrosophic graph $L(\mathbb{G}) = (A_2, B_2)$ such that

(i) $T_{A_2}(S_x) = T_{B_1}(x) = T_{B_1}(u_x v_x)$,
$I_{A_2}(S_x) = I_{B_1}(x) = I_{B_1}(u_x v_x)$,
$F_{A_2}(S_x) = F_{B_1}(x) = F_{B_1}(u_x v_x)$,

(ii) $T_{B_2}(S_x S_y) = \min\{T_{B_1}(x), T_{B_1}(y)\}$,
$I_{B_2}(S_x S_y) = \min\{I_{B_1}(x), I_{B_1}(y)\}$,
$F_{B_2}(S_x S_y) = \max\{F_{B_1}(x), F_{B_1}(y)\}$ for all $S_x, S_y \in C, S_x S_y \in D$.

Proposition 2.33. $L(\mathbb{G}) = (A_2, B_2)$ *is a neutrosophic line graph of some neutrosophic graph* $\mathbb{G} = (A_1, B_1)$ *if and only if*

$$
\begin{aligned}
T_{B_2}(S_x S_y) &= \min\{T_{A_2}(S_x), T_{A_2}(S_y)\}, \\
I_{B_2}(S_x S_y) &= \min\{I_{A_2}(S_x), T_{A_2}(S_y)\}, \\
F_{B_2}(S_x S_y) &= \max\{F_{A_2}(S_x), F_{A_2}(S_y)\},
\end{aligned}
$$

for all $S_x S_y \in D$.

Definition 2.34. Let $\mathbb{H} = (V, \mathbb{E})$ be a neutrosophic hypergraph of a simple graph $H = (V, E^*)$, and $L(H) = (X, \varepsilon)$ be a line graph of H. The *neutrosophic line graph* $L(\mathbb{H})$ *of a neutrosophic hypergraph* $\mathbb{H}$ is defined to be a pair $L(\mathbb{H}) = (A, B)$, where A is the vertex set of $L(\mathbb{H})$ and B is the edge set of $L(\mathbb{H})$ as follows:

(i) A is a neutrosophic set of X such that
$T_A(E_i) = \max_{v \in E_i}(T_{E_i}(v))$,
$I_A(E_i) = \max_{v \in E_i}(I_{E_i}(v))$,
$F_A(E_i) = \min_{v \in E_i}(F_{E_i}(v))$ for all $E_i \in \mathbb{E}$,

(ii) B is a neutrosophic set of ε such that
$T_B(E_jE_k) = \min_i\{\min(T_{E_j}(v_i), T_{E_k}(v_i))\}$,
$T_B(E_jE_k) = \min_i\{\min(T_{E_j}(v_i), T_{E_k}(v_i))\}$,
$T_B(E_jE_k) = \max_i\{\max(T_{E_j}(v_i), T_{E_k}(v_i))\}$, where $v_i \in E_i \cap E_j$, $j, k = 1, 2, 3, \cdots, n$.

Example 2.35. Consider $H = (V, E^*)$; $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E^* = \{E_1, E_2, E_3, E_4, E_5, E_6\}$, where $E_1 = \{v_1, v_3\}, E_2 = \{v_1, v_2\}, E_3 = \{v_2, v_4\}$, $E_4 = \{v_3, v_6\}$, $E_5 = \{v_3, v_5\}$, $E_6 = \{v_5, v_6\}$, and $E_7 = \{(v_4, v_6\}$, $\mathbb{H} = (V, \mathbb{E})$ as $\mathbb{E} = \{E_1, E_2, E_3, E_4, E_5, E_6\}$, such that
$E_1 = \{(v_1, 0.3, 0.4, 0.6), (v_3, 0.7, 0.4, 0.4)\}, E_2 = \{(v_1, 0.3, 0.4, 0.6), (v_2, 0.5, 0.7, 0.6)\}$,
$E_3 = \{(v_2, 0.5, 0.7, 0.6), (v_4, 0.6, 0.4, 0.8)\}$, $E_4 = \{(v_3, 0.7, 0.4, 0.4), (v_6, 0.4, 0.2, 0.7)\}$,
$E_5 = \{(v_3, 0.7, 0.4, 0.4), (v_5, 0.6, 0.7, 0.5)\}$, $E_6 = \{(v_5, 0.6, 0.7, 0.5), (v_6, 0.4, 0.2, 0.7)\}$,
$E_7 = \{(v_4, 0.6, 0.4, 0.8), (v_6, 0.4, 0.2, 0.7)\}$.
The neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ is shown in Figure. 2.2.

The line graph $L(\mathbb{H})$ of neutrosophic hyperraph $\mathbb{H}$ is $L(\mathbb{H}) = (A, B)$, where
$A = \{(E_1, 0.7, 0.4, 0.4), (E_2, 0.5, 0.7, 0.6), (E_3, 0.6, 0.7, 0.6), (E_4, 0.7, 0.4, 0.4), (E_5, 0.7, 0.7, 0.4), (E_6, 0.6, 0.7, 0.5), (E_7, 0.6, 0.4, 0.7)\}$ is the vertex set and
$B = \{(E_1E_2, 0.3, 0.4, 0.6), (E_1E_5, 0.7, 0.4, 0.4), (E_1E_4, 0.7, 0.4, 0.4), (E_2E_3, 0.5, 0.7, 0.6), (E_3E_7, 0.6, 0.4, 0.8), (E_4E_5, 0.7, 0.4, 0.4), (E_4E_6, 0.4, 0.2, 0.7), (E_4E_7, 0.4, 0.2, 0.7), (E_5E_6, 0.6, 0.7, 0.5), (E_6E_7, 0.4, 0.2, 0.7)\}$
is the edge set of the neutrosophic line graph of $\mathbb{H}$.

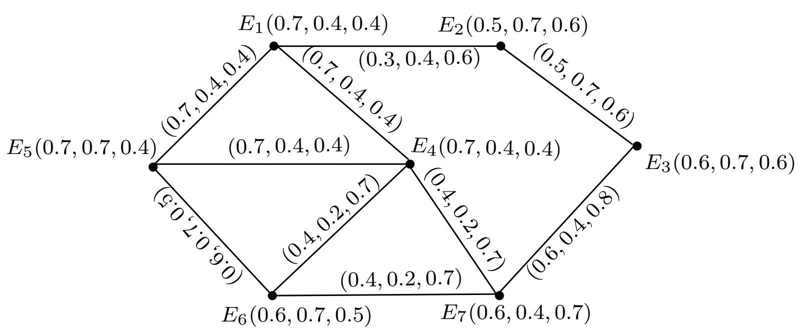

Figure 2.4: Neutrosophic line graph $L(\mathbb{H})$ of neutrosophic hypergraph $\mathbb{H}$.

Proposition 2.36. *A neutrosophic hypergraph is connected if and only if line graph of an hypergraph is connected.*

Definition 2.37. A $2-$section of neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, denoted by $[\mathbb{H}]_2$, is a neutrosophic graph $\mathbb{G} = (A, B)$, where A is the neutrosophic vertex of V, B is the neutrosophic edge set in which any two vertices form an edge if they are in the same neutrosophic hyperedge such that

$$\begin{aligned}
T_B(e) &= \min\{T_{E_k}(v_i), T_{E_k}(v_j)\},\\
I_B(e) &= \min\{I_{E_k}(v_i), T_{E_k}(v_j)\},\\
F_B(e) &= \max\{F_{E_k}(v_i), F_{E_k}(v_j)\},
\end{aligned}$$

for all $E_k \in \mathbb{E}, i \neq j, k = 1, 2, \cdots, m$.

Definition 2.38. The dual of a neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$ is a neutrosophic hypergraph $\mathbb{H}^* = (E, \mathbb{V})$; $E = \{e_1, e_2, \cdots, e_n\}$ set of vertices corresponding to $E_1, E_2, , \cdots, E_n$ respectively and $\mathbf{V} = \{V_1, V_2, \cdots, V_n\}$ set of hyperedges corresponding to $v_1, v_2, \cdots, v_n$ respectively.

Example 2.39. Let $\mathbb{H} = (V, \mathbb{E})$ be an neutrosophic hypergraph such that $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $\mathbb{E} = \{E_1, E_2, E_3\}$, where $E_1 = \{(v_1, 0.5, 0.4, 0.6), (v_2, 0.4, 0.3, 0.8)\}$, $E_2 = \{(v_2, 0.4, 0.3, 0.8), (v_3, 0.6, 0.4, 0.8), (v_4, 0.7, 0.4, 0.5)\}$, and $E_3 = \{(v_4, 0.7, 0.4, 0.5), (v_5, 0.4, 0.2, 0.9)\}$.

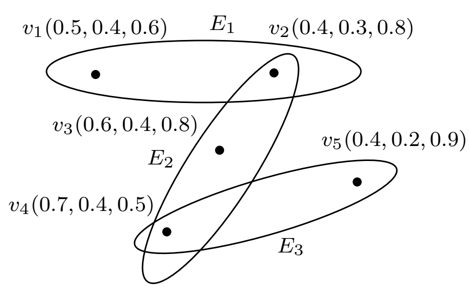

Figure 2.5: Neutrosophic hypergraph.

The Neutrosophic hypergraph can be represented by the following incidence matrix:

$M_{\mathbb{H}}$	E_1	E_2	E_3
v_1	$(0.5, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$
v_2	$(0.4, 0.3, 0.8)$	$(0.4, 0.3, 0.8)$	$(0, 0, 0)$
v_3	$(0, 0, 0)$	$(0.6, 0.4, 0.8)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0.7, 0.4, 0.5)$	$(0.7, 0.4, 0.5)$
v_5	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.9)$

The dual neutrosophic hypergraph $\mathbb{H}^* = (E, \mathbb{V})$ of $\mathbb{H}$ such that $E = \{e_1, e_2, e_3\}$, $\mathbb{V} = \{V_1, V_2, V_3, V_4, V_5\}$, where $V_1 = \{(e_1, 0.5, 0.4, 0.6), (e_2, 0, 0, 0), (e_3, 0, 0, 0)\}$, $V_2 = \{(e_1, 0.4, 0.3, 0.8), (e_2, 0.4, 0.3, 0.8), (e_3, 0, 0, 0)\}$, $V_3 = \{(e_1, 0, 0, 0), (e_2, 0.6, 0.4, 0.8), (e_3, 0, 0, 0)\}$, $V_4 = \{(e_1, 0, 0, 0), (e_2, 0.7, 0.4, 0.5), (e_3, 0.7, 0.4, 0.5)\}$ and $V_5 = \{(e_1, 0, 0, 0), (e_2, 0, 0, 0), (e_3, 0.4, 0.2, 0.9)\}$.

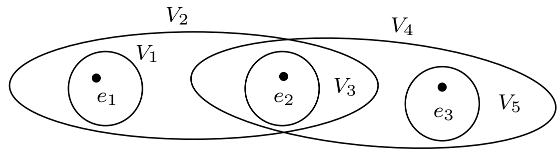

Figure 2.6: Dual neutrosophic hypergraph $\mathbb{H}^*$.

The dual neutrosophic hypergraph is shown in Figure. 2.6 and its incidence matrix $M_{\mathbb{H}^*}$ is as follows:

$M_{\mathbb{H}^*}$	V_1	V_2	V_3	V_4	V_5
e_1	$(0.5, 0.4, 0.6)$	$(0.4, 0.3, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
e_2	$(0, 0, 0)$	$(0.4, 0.3, 0.8)$	$(0.6, 0.4, 0.8)$	$(0.7, 0.4, 0.5)$	$(0, 0, 0)$
e_3	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.7, 0.4, 0.5)$	$(0.4, 0.2, 0.9)$

Remark 2.40. Incidence matrix of $\mathbb{H}^*$ is the transpose of the incidence matrix of neutrosophic hypergraph $\mathbb{H}$ and $\triangle(\mathbb{H}) = rank(\mathbb{H}^*)$. The dual neutrosophic hypergraph $\mathbb{H}^*$ of a simple neutrosophic hypergraph $\mathbb{H}$ may or may not be simple.

Proposition 2.41. *The dual neutrosophic hypergraph $\mathbb{H}^*$ of a linear neutrosophic hypergraph $\mathbb{H}$ without isolated vertex is linear neutrosophic hypergraph.*

Proof. Let $\mathbb{H}$ be a linear neutrosophic hypergraph. Assume that $\mathbb{H}^*$ is not linear neutrosophic hypergraph, then there exist two distinct neutrosophic hyperedges V_i and V_j of $\mathbb{H}^*$ have at least two vertices e_1 and e_2 in common. By defnition of dual neutrosophic hypergraph implies that v_i and v_j belongs to E_1 and E_2 (the neutrosophic hyperedges of $\mathbb{H}$ standing for the vertices e_1, e_2 of $\mathbb{H}^*$, respectively) so $\mathbb{H}$ is not linear neutrosophic hypergraph. Contradiction since $\mathbb{H}$ is linear neutrosophic hypergraph. Hence dual $\mathbb{H}^*$ of a linear neutrosophic hypergraph without isolated vertex is also linear neutrosophic hypergraph. $\square$

Observation In crisp theory of hypergraphs, any non-trivial graph is the line graph of a linear hypergraph and line graph of a hypergraph is the $2-$section of dual hypergraph, but a neutrosophic graph is not necessarly the line graph of a linear neutrosophic hypergraph and neutrosophic line graph of hypergraph is not the $2-$section of dual neutrosophic hypergraph.

Definition 2.42. Let $\mathbb{H} = (V, \mathbb{E})$ be a neutrosophic hypergraph. A *neutrosophic transversal* τ of $\mathbb{H}$ is a neutrosophic subset of V such that $\tau^{h(E_i)} \cap E^{h(E_i)} \neq \emptyset$ for all $E_i \in \mathbb{E}$, where $h(E_i)$ is the height of neutrosophic hyperedge E_i. A minimal neutrosophic transversal τ for $\mathbb{H}$ is a transversal of $\mathbb{H}$ if $\tau' \subset \tau$, then τ' is not a neutrosophic transversal of $\mathbb{H}$.

Proposition 2.43. *If τ is a neutrosophic transversal of a neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, then $h(\tau) > h(E_i)$ for all $E_i \in \mathbb{E}$, and if τ is a minimal neutrosophic transversal of $\mathbb{H}$, then $h(\tau) = \vee\{h(E_i) \mid E_i \in \mathbb{E}\} = h(\mathbb{H})$.*

Theorem 2.44. *For a neutrosophic hypergraph $\mathbb{H}$, $Tr(\mathbb{H}) \neq \emptyset$, where $Tr(\mathbb{H})$ denotes the family of minimal neutrosophic transversals of $\mathbb{H}$.*

Proposition 2.45. *For neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, the following statements are equivalent:*

(i) *τ is a neutrosophic transversal of $\mathbb{H}$*

(ii) *For every $E_i \in \mathbb{E}$, $h(E_i) = (r', s', t')$, and each $0 < r \leq r', 0 < s \leq s', t \geq t'$, $\tau^{(r,s,t)} \cap E_i^{(r,s,t)} \neq \emptyset$*

If the $(r, s, t)-$cut $\tau^{(r,s,t)}$ is a subset of $V^{(r,s,t)}$ for all (r, s, t), then

(iii) *For each (r, s, t), $0 < r \leq r', 0 < s \leq s', t \geq t'$, $\tau^{(r,s,t)}$ is a neutrosophic transversal of $\mathbb{H}^{(r,s,t)}$*

(iv) *Each neutrosophic transversal τ of $\mathbb{H}$ contains a neutrosophic transversal τ' for each (r, s, t), $0 < r \leq r', 0 < s \leq s', t \geq t'$, $\tau'^{(r,s,t)}$ is a transversal of $H^{(r,s,t)}$*

Observation: If τ is a minimal neutrosophic transversal of neutrosophic graph $\mathbb{H}$, then $\tau^{(r,s,t)}$ not necessarily belongs to $Tr(\mathbb{H}^{(r,s,t)}$ for each (r, s, t), satisfying $0 < r \leq r', 0 < s \leq s', t \geq t'$. Let $Tr^*(\mathbb{H})$ represents the collection of those minimal neutrosophic transversal, τ of $\mathbb{H}$, where $\tau^{(r,s,t)}$ is a minimal neutrosophic transversal of $\mathbb{H}^{(r,s,t)}$, for each (r, s, t), $0 < r \leq r', 0 < s \leq s', t \geq t'$, i.e., $Tr^* = \{\tau \in Tr(\mathbb{H}) \mid h(\tau) = h(\mathbb{H}) \text{ and } \tau^{(r,s,t)} \in Tr(\mathbb{H}^{(r,s,t)})\}$.

Example 2.46. Consider the neutrosophic hypergraph $\mathbb{H} = (V, \mathbb{E})$, where $V = \{v_1, v_2, v_3\}$ and $\mathbb{E} = \{E_1, E_2, E_3\}$, which is represented by the following incidence matrix:

$M_{\mathbb{H}}$	E_1	E_2	E_3
v_1	$(0.9, 0.6, 0.1)$	$(0, 0, 0)$	$(0.4, 0.3, 0.2)$
v_2	$(0.4, 0.3, 0.2)$	$(0.4, 0.3, 0.2)$	$(0.4, 0.3, 0.2)$
v_3	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.3, 0.2)$

Clearly, $h(\mathbb{H}) = 0.9$, the only minimal transversal τ of neutrosophic hypergraph $\mathbb{H}$ is $\tau(\mathbb{H}) = \{(v_1, 0.9, 0.6, 0.1), (v_2, 0.4, 0.3, 0.2)\}$. F($\mathbb{H}$) of $\mathbb{H}$ is F($\mathbb{H}$) $= \{(0.9, 0.6, 0.1), (0.4, 0.3, 0.2)\}$, $\tau^{(0.9,0.6,0.1)} = \{a\}$ and $\tau^{(0.4,0.3,0.2)} = \{a, b\}$. Since $\{b\}$ is the only minimal neutrosophic transversal of the $\mathbb{H}^{(0.4,0.3,0.2)}$, $E^{(0.4,0.3,0.2)} = \{\{v_1, v_2\}, \{v_2\}, \{v_1, v_2, v_3\}\}$, it follows that the only minimal transversal τ of $\mathbb{H}$ is not a member of $Tr^*(\mathbb{H})$. Hence $Tr^*(\mathbb{H}) = \emptyset$.

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